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DETERMINATION OF THE HYDRAULIC RESISTANCE OF A REGENERATOR'S PACKING WITH THE FINITE-ELEMENT METHOD

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A technique for determining theoretically the hydraulic resistance of a regenerator's packing has been considered. This technique is based on the finite-element method and may be used in the step of designing of the regenerator where it is necessary to determine the main parameters of the packing without resorting to full-scale tests.

Owing to the development of digital technologies it has become possible to improve the quality of calculation of regenerators used in air-separating units and in cold-air refrigerating machines by increasing the number of factors in question. However, realization of all of the advantages of automatic-design systems requiress a complete database of the properties of substances modeled in the considered range of temperatures and pressures and a program making it possible to process these data.

Most thermodynamic and hydraulic calculations are based on similarity theory. The processes in an object are represented using a system of criterial equations. As a result, calculation is carried out by analogy with the existing model or according to the results of testing a specimen.

Below, we give a procedure that enables us to quite easily perform computations, assigning just the external factors.

In calculating such heat-exchange apparatuses as regenerators [1], we must have their characteristic dimensions (diameter and height), the type of packing, the mean temperature and pressure of the gas, etc. There is a well-studied type-dimension series of packings (most efficient in operation) [2] for their calculation. The adoption of automatic-design systems for processing of data and computer-aided solution of problems according to the procedures presented in [3, 4] has made it possible to raise the productivity and to improve the quality of the work carried out. Since most factors have been taken into account in numerical experiments, and corrections for actual conditions enable one to make the error insignificant, the corresponding calculations yield satisfactory results. However, calculation of a regenerator with another type of packing (new or modernized) may cause a significant increase in the error, and such computations are simply unacceptable in some cases. To solve this problem one should manufacture a specimen and carry out the corresponding tests; however, such an approach may take a good deal of time.

Owing to the significant increase in the rate (number of operations) and depth (use of sixty-four-bit numbers) of computer-aided calculation, most of the work on "manufacture," testing, and operational development of an object may be carried out theoretically. For this purpose we simulate the processes of interest on a three-dimensional model which is a precise replica (or a model to a scale convenient for work) of the taken specimen of the regenerator's packing. Graphics editors, e.g., AutoCAD, SolidWorks, 3D S Max, Kompas, and others, are conveniently used for creation of models of an object.

Creating a model directly in a graphic version is a fairly long (up to a few weeks) and not entirely exact process. The model may be created most rapidly and qualitatively using a mathematical description of its boundaries.

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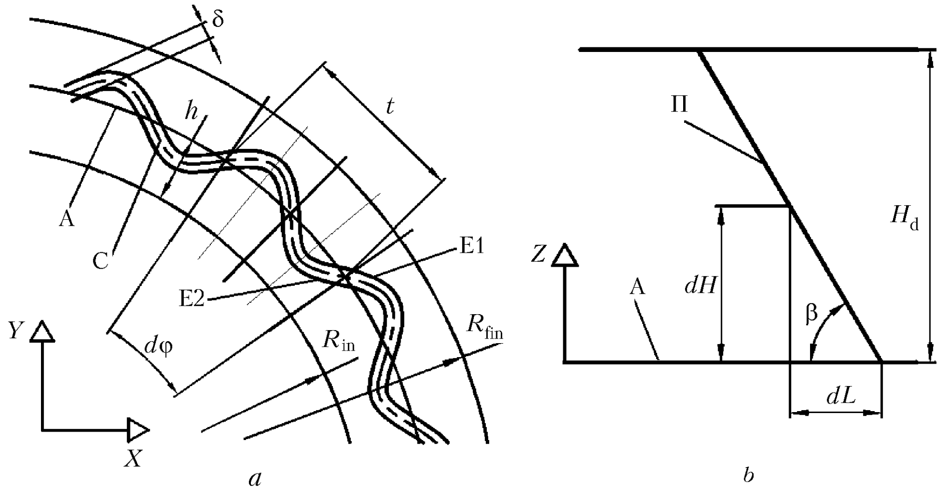


Fig. 1. Element of a corrugated band with characteristic dimensions: a) in the *XY* plane; b) along the *Z* axis.

Two specimens must be manufactured for experiments when a radically new structure is introduced. One specimen is used for additional description of the processes or properties that are beyond the scope of the earlier tests, whereas the other, the control one, is used to confirm theory.

The finite-element (volume) method is the most accurate. It is based on system disintegration of the model to form elements (cells) in which the processes may be described by simple dependences (linear laws) and the parameters may be taken to be constant. The method is universal and encompasses a wide range of problems of hydrodynamics, heat transfer, electrodynamics, strength of materials, chemistry, and many others. An advantage of the method is the integration of the model's geometry directly into the process of calculation, which gives an error lower than 5%, when the basic processes are considered [5, 6].

To efficiently distribute computer time it is sufficient to manufacture a few models that will represent all of the features of an object most accurately. For example, the packing of the regenerator contains a few elements (corrugations or basalt grit) extending with no changes over the entire volume. Subsequently, the data obtained characterizing, e.g., hydraulic resistance, may be included in the similarity-based calculation of the regenerator. Thus, for creation of a three-dimensional model of a metal packing in the Cartesian coordinate system, we use the mathematical dependences and data given in [7].

Figure 1a shows a portion of a corrugated band. The corrugation profile is formed by two curves, E1 and E2, which are the equidistances to curve C and are offset by half the band thickness δ from it. In the cross section *XY*, we represent a disk from a corrugated band as a sinusoid with an axis corresponding to the Archimedean spiral. The displacement of corrugations along the *Z* axis (Fig. 1b) corresponds to the angle of inclination β of the corrugations, and their axis is located on the same spiral.

A regular sinusoid located, e.g., in the *XY* plane (Fig. 1a), has the form $y = \sin x$ [7]. For the sine curve to correspond to the geometric dimensions of the packing, we introduce a grooving pitch t and a grooving height h. This yields

$$y = \frac{h - \delta}{2} \sin\left(\frac{2\pi}{t} x\right). \tag{1}$$

We obtain equidistant curves with the use of a set of points, constructing perpendiculars of size $\delta/2$ to the tangents to the curve (1). The slope of the curve (1) is its derivative at this point:

$$\tan \alpha = \frac{h - \delta}{2} \pi \cos\left(\frac{2\pi}{t} x\right).$$

On the same plane, curves E1 and E2 will have the form

$$\text{E1}: \begin{cases} x^{\text{E1}} = x + \dfrac{\delta}{2}\cos\alpha, \\ y^{\text{E1}} = y + \dfrac{\delta}{2}\sin\alpha, \end{cases} \quad \text{E2}: \begin{cases} x^{\text{E2}} = x - \dfrac{\delta}{2}\cos\alpha, \\ y^{\text{E2}} = y - \dfrac{\delta}{2}\sin\alpha. \end{cases}$$

Substituting here the values of the function $y(x)$ and the slope α, we obtain

$$\text{E1}: \begin{cases} x^{\text{E1}} = x + \dfrac{\delta}{2}\cos\left(\arctan\left(\dfrac{h-\delta}{t}\pi\cos\left(\dfrac{2\pi}{t}x\right)\right)\right), \\ y^{\text{E1}} = \dfrac{h-\delta}{2}\sin\left(\dfrac{2\pi}{t}x\right) + \dfrac{\delta}{2}\sin\left(\arctan\left(\dfrac{h-\delta}{t}\pi\cos\left(\dfrac{2\pi}{t}x\right)\right)\right), \end{cases} \tag{2}$$

$$\text{E2}: \begin{cases} x^{\text{E2}} = x - \dfrac{\delta}{2}\cos\left(\arctan\left(\dfrac{h-\delta}{t}\pi\cos\left(\dfrac{2\pi}{t}x\right)\right)\right), \\ y^{\text{E2}} = \dfrac{h-\delta}{2}\sin\left(\dfrac{2\pi}{t}x\right) - \dfrac{\delta}{2}\sin\left(\arctan\left(\dfrac{h-\delta}{t}\pi\cos\left(\dfrac{2\pi}{t}x\right)\right)\right). \end{cases} \tag{3}$$

In our case, the axis of these curves corresponds to the Archimedean spiral A [7] (Fig. 1). The length of the latter relative to the angle of rotation φ for the condition of close winding of the band is described by the equation

$$L = \frac{h}{4\pi}\left(\varphi\sqrt{\varphi^2+1} + \operatorname{arsinh}\varphi\right), \tag{4}$$

and the spiral radius is determined as

$$R = \frac{h}{2\pi}\varphi. \tag{5}$$

Comparing Eqs. (1), (4), and (5), substituting the Archimedean-spiral length L for x in (1), and reducing the resulting equation to the axes shown in Fig. 1a, we obtain a system of equations for curve C:

$$\text{C}: \begin{cases} x = \left(R + \dfrac{h-\delta}{2}\sin\left(\dfrac{2\pi}{t}L\right)\right)\cos\varphi, \\ y = \left(R + \dfrac{h-\delta}{2}\sin\left(\dfrac{2\pi}{t}L\right)\right)\sin\varphi. \end{cases} \tag{6}$$

Analogously we may represent the equations of equidistances E1 and E2.

Figure 1b is a diagrammatic representation of the trajectory Π of displacement of the corrugation cross section along the Z axis. This trajectory may be subdivided into a few portions with a step dL along the Archimedean spiral A and a step dH along the disk height H_d. The ratio of these steps corresponds to the slope of corrugations β and may be written as

$$dH/dL = \tan\beta. \tag{7}$$

The three-dimensional model is easily created using Eqs. (2), (3), (6), and (7) and the initial geometric parameters. For calculations we use the finite-element method, which enables us to solve a range of various problems and is capable of replacing full-scale tests. An example of such programs is provided by ANSYS, COSMOS, and others.

Example. The hydraulic resistance of a packing is calculated as follows. In the case in question, we model a packing manufactured from a corrugated aluminum band [8]: $\beta = 60^{\circ}$, $t = 3.14\cdot10^{-3}$ m, $h = 1.35\cdot10^{-3}$ m, $\delta = 0.4\cdot10^{-3}$ m, and $H_d = 0.04$ m. We model an angular sector in $d\varphi = 10^{\circ}$ for $R_{in} = 0.405$ m, $R_{fin} = 0.430$ m, and $H_d = 0.04$ m (Fig. 1). The dimensions of the sector have been selected from the condition of minimum influence of the boundaries

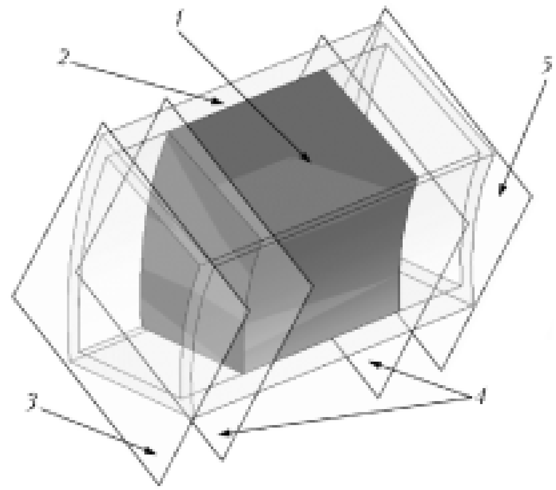
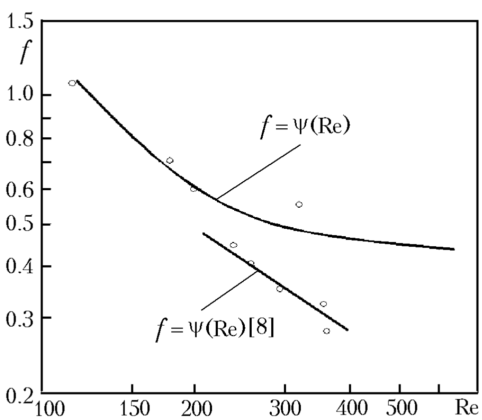


Fig. 2. Diagram of the model of the regenerator's packing: 1) element of the regenerator's packing; 2) channel for passage of the gas; 3) inlet cross section of the flow; 4) control planes (on the packing ends); 5) outlet cross section of the flow.



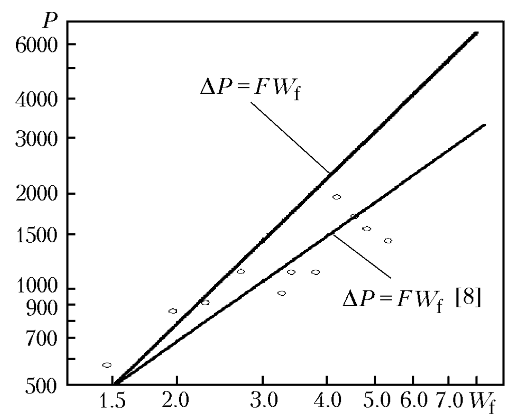


Fig. 3. Change in the coefficient of hydraulic resistance as a function of the Reynolds number. *f*, 1/m

Fig. 4. Change in the pressure loss by one meter of packing as a function of the mass velocity. *P*, Pa; W_f, kg/(m^3·sec).

on the processes in the packing's element. With mathematical dependences (2) and (3) whose axis corresponds to the spiral A, we describe the packing's element by Eq. (6) and create its three-dimensional model using Eq. (7). Regenerators of air-separating units and cold-air refrigerating machines in a regular version have three pronounced zones: a hot zone, a moderate zone, and a cold one. They are formed due to the change in the free volume: it is the largest in the hot zone and the smallest in the cold one. The size of the free volume is determined by the geometric dimensions of the packing. In this case, we have considered just one zone of the regenerator's packing, which corresponds to the geometric parameters given above.

Using the editors of the program (e.g., COSMOS), we specify the time, physical, geometric, and boundary conditions on the model. The operating regime of the regenerator is steady-state with parameters constant with time. We use air as the medium for calculation and aluminum as the packing material. The geometric data are assigned directly by the model. We have heat exchange between the environment and the body's surface (boundary conditions of the fourth kind).

We construct a channel for passage of the gas through the packing's element (Fig. 2). Thereafter we assign two cross sections at a certain distance from the packing: the inlet and outlet cross sections. They must be arranged

TABLE 1. Results of Calculation for 1 m of Packing Length

W_f	Re	f	ΔP	
			of one disk	per 1 m of packing
1.5	118.0	1.0757	22.58	564.6
2.0	157.4	0.7321	27.32	683.0
3.0	236.0	0.5424	45.54	1138.5
4.0	314.7	0.4907	73.25	1831.4
5.0	393.4	0.4662	108.8	2718.9
6.0	472.0	0.4520	151.8	3795.6
7.0	550.7	0.4414	201.8	5044.5
8.0	629.4	0.4346	259.5	6487.8

so that the flows are uniformly distributed throughout the volume; otherwise, an additional error disturbing the course of the process will appear in calculation.

We assign the velocity vector ($\mathbf{W}_f$) and the temperature (T = 290 K) for the incoming flow and the pressure (P = 0.1 MPa) for the outgoing flow. We assign a uniform change in the temperature with packing height (we assume that the underrecuperation is equal to ΔT = 5 K and the initial value of the temperature difference along the disk height is ΔT = 5 K). The calculation is carried out without allowance for the deposition of impurities (water and carbon dioxide) in the packing's element. Next we obtain the pressure difference in the assigned cross sections (here these are the boundaries of the packing of height H_d = 0.04 m), which is the hydraulic resistance of the packing. The results obtained are given in Table 1 and are presented in Figs. 3 and 4.

Table 1 gives the calculated values of the Reynolds number

$$\mathrm{Re} = \frac{w_f d_e \rho}{\mu}, \tag{8}$$

the coefficient of hydraulic resistance

$$f = \frac{2\Delta P d_e}{\rho w_f} = \frac{8\Delta P e}{\rho w_f S}, \tag{9}$$

where the rate of filtration w_f of the gas through the packing is calculated from the formula $w_f = W_f/\rho$, and the pressure loss, which have been obtained in modeling.

Figure 3 gives the coefficient of hydraulic resistance as a function of the Reynolds number. For the sake of comparison, we have applied points that were obtained as a result of the experiments conducted with an analogous packing in [8]. Also, this figure gives the curve characterizing the change in the coefficient of hydraulic resistance as a function of the Reynolds number [8]. The curve obtained is characteristic of regenerator packings with a corrugated band. We observe a laminar flow regime characterized by a rectilinear portion (Re < 250) and a bent transient portion of the curve with further turbulization of the flow.

Figure 4 plots the pressure loss as a function of the mass velocity. Points correspond to the data obtained as a result of the experiments [8], with an analogous packing. The results obtained are comparable to the data of [8], where calculations and experiment for an analogous disk were carried out. As follows from the figure, the pressure loss uniformly increases with flow velocity and has a rectilinear dependence, just as in [8].

In Figs. 3 and 4, it is seen that experimental points (particularly in the laminar regime of flow) fairly well coincide with calculated curves. This suggests that this form of modeling may be recommended for evaluation of the hydraulic resistance of the regenerator's packing.

The slightly overstated pressure loss and coefficient of hydraulic resistance compared to the data obtained in [8] for an analogous geometry of the packing may be explained as follows:

1) in the case in question we have considered an individual disk, whereas in [8], consideration was given to the entire regenerator where the disks are spaced a certain distance (not indicated in [8]) apart; in this connection, the pressure loss was given without allowance for the interdisk clearance;

2) in modeling, we observed the phenomenon of reflection of a part of the straight flow from the end surfaces of the corrugated band, which produced additional resistance to the motion and turbulization of the flow;

3) calculation was carried out for one disk the temperature difference in which was 5 K, unlike the temperature difference on a larger portion of the packing in [8] (it is about 150 K for the total height of the regenerator and 50 K for one zone of the disks).

The time of creation of the model in AutoCAD and SolidWorks was 3 min (with an established accuracy of 12 characters). The time of recognition of the model with the creation of more than 100 thousand cells was 20 min. Calculation of the "mean accuracy" takes 15 min and attains about 150 iterations. Two Intel Xeon processors operating in parallel with a frequency of 2.8 GHz have been used.

CONCLUSIONS

1. Despite the fairly long computational time (about an hour per packing element), this method of designing is more rapid than full-scale tests.

2. The method of modeling proposed for determination of the hydraulic resistance of the regenerator's packing makes it possible to carry out calculation for different variants of the packing geometry with the aim of selecting the most optimum parameters, for which the hydraulic resistance is minimum.

NOTATION

dH, step along the disk height, m; dL, step along the Archimedean spiral, m; d_e, equivalent diameter, m; $d\varphi$, angle of the circular sector of the packing element, deg; e, free volume, m^3/m^3; $F()$, function of the dependence of the pressure loss on the mass velocity; f, coefficient of hydraulic resistance, 1/m; H_d, disk height, m; h, grooving height, m; L, length, m; P, pressure, Pa; R, radius, m; Re, Reynolds number; R_{fin} and R_{in}, final and initial radii, m; S, specific surface, m^2/m^3; T, temperature, K; t, grooving pitch, m; W_f, mass rate of filtration of the gas, referred to the flow area of the packing under normal conditions (P = 101,325 Pa and T = 273.15 K), kg/(m^2·sec); w_f, filtration rate, m/sec; XYZ, Cartesian coordinate system; x, y, and z, values of the coordinates; α, slope of the curve at an assigned point to the horizon, deg; β, angle of inclination of corrugations, deg; ΔP, hydraulic resistance, Pa; ΔT, temperature difference, K; δ, band thickness, m; μ, dynamic viscosity of the gas at the mean temperature and pressure, Pa·sec; ρ, density of the gas at the mean temperature and pressure, kg/m^3; φ, angle of rotation of a point in polar coordinates, rad; $\psi()$, function of the dependence of the coefficient of hydraulic resistance on the Reynolds number. Subscripts: d, disk; fin, final; in, initial; f, filtration; e, equivalent.

REFERENCES

1. S. S. Budnevich, *The Processes of Deep Cooling* [in Russian], Moscow–Leningrad (1966).
2. A. Bartsch, *Regeneratoren des Tieftempaturtechnik*, Veb Verlagtechni, Berlin (1962).
3. F. M. Pruchkina and M. B. Stolper, Calculation of regenerators of air-separating devices on a digital computer, *Khim. Neft. Mashinostr.*, No. 1, 14–18 (1970).
4. V. P. Alekseev, G. E. Vainshtein, and P. V. Gerasimov, *Calculation and Modeling of Cryogenic Apparatuses* [in Russian], Énergoatomizdat, Leningrad (1987).
5. P. Kohnke (Ed.), "ANSYS Users Manual. 001369. Twelfth Edition," ANSYS, Inc., Southpointe 275 Technology Drive, Canonsburg, PA, ansysinfo@ansys.com (2001).
6. Fluid Flow Simulation software. Cosmos Flo Users Theory Manual. Structural Research and Analysis Corporation (SRAC). www.cosmosm.com.
7. I. N. Bronshtein and K. A. Semendyaev, *Handbook of Mathematics for Engineers and Students of Higher Technical School* [in Russian], 3rd edn., GITTL, Moscow (1953).
8. N. K. Elukhin and O. I. Starosvitskii, Heat transfer and hydraulic drag in disk packings in regenerators, in: *Apparatuses and Machines of Oxygen Installations* [in Russian], Issue 7, Mashgiz, Moscow (1963), pp. 73–89.